## In the Claims

Claim 1 (canceled)

Claim 2 (previously presented): The system estimation method according to claim 7, wherein the processing section calculates the existence condition in accordance with a following expression:

$$\hat{\boldsymbol{\Sigma}}_{i|i}^{-1} = \hat{\boldsymbol{\Sigma}}_{i|i-1}^{-1} + \frac{1 - \gamma_f^{-2}}{\rho} \boldsymbol{H}_i^T \boldsymbol{H}_i > 0, \quad i = 0, \dots, k$$
 (17)

Claim 3 (previously presented): The system estimation method according to claim 7, wherein the processing section calculates the existence condition in accordance with a following expression:

$$-\rho \hat{\Xi}_{i} + \rho \gamma_{f}^{2} > 0, \quad i = 0, \dots, k$$

$$\rho = 1 - \gamma_{f}^{2}, \quad \hat{\Xi}_{i} = \frac{\rho \boldsymbol{H}_{i} \boldsymbol{K}_{s,i}}{1 - \boldsymbol{H}_{i} \boldsymbol{K}_{s,i}}, \quad \rho = 1 - \chi(\gamma_{f})$$

$$(18)$$

where the forgetting factor  $\rho$  and the upper limit value  $\gamma_f$  have a following relation:

 $0<\rho=1-\chi(\gamma_f)\leq 1$ , where  $\chi(\gamma_f)$  denotes a monotonically damping function of  $\gamma_f$  to satisfy  $\chi(1)=1$  and  $\chi(\infty)=0$ .

Claims 4-6 (canceled)

Claim 7 (currently amended): A system estimation method, for a communication system or s sound system or sound field reproduction or noise control, for making state estimation

robust and optimizing a forgetting factor  $\boldsymbol{\rho}$  simultaneously in an estimation algorithm, in which

for a state space model expressed by following expressions:

 $x_{k+1} = F_k x_k + G_k w_k$ 

 $y_k = H_k x_k + v_k$ 

 $z_k = H_k x_k$ 

here,

 $x_k$ : a state vector or simply a state,

 $w_k$ : a system noise,

v<sub>k</sub>: an observation noise,

yk: an observation signal,

 $z_k$ : an output signal,

 $\ensuremath{\mathbb{F}_k} \colon$  dynamics of a system, and

 $G_k$ : a drive matrix,

as an evaluation criterion, a maximum value of an energy gain which indicates a ratio of a filter error to a disturbance including the system noise  $w_k$  and the observation noise  $v_k$  and is weighted with the forgetting factor  $\rho$  is suppressed to be smaller than a term corresponding to a previously given upper limit value  $\gamma_f,$  and

the system estimation method comprises:

a step at which a processing section inputs the upper limit value  $\gamma_f$ , the observation signal  $y_k$  as an input of a filter and a value including an observation matrix  $H_k$  from a storage section or an input section;

a step at which the processing section determines the forgetting factor  $\rho;$  as a following function of  $\gamma_f,$   $\rho{=}1{-}\chi(\gamma_f)$ 

where  $\chi(\gamma_f)$  denotes a monotonically damping function of  $\gamma_f$  to satisfy  $\chi(1)$  = 1 and  $\chi(\infty)$  = 0,;

a step of executing a hyper  $H_{\infty}$  filter at which the processing section reads out an initial value or a value

including the observation matrix  $H_k$  at a time from the storage section and obtains a filter gain  $K_{s,k}$  by using the forgetting factor  $\rho$  and a gain matrix  $K_k$  and by following expressions (20) to (22):

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k-1|k-1} + \boldsymbol{K}_{s,k} (y_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k-1|k-1})$$
(20)

$$K_{s,k} = K_k(:,1)/R_{e,k}(1,1)$$
,  $K_k = \rho^{\frac{1}{2}}(\rho^{-\frac{1}{2}}K_kR_{e,k}^{-\frac{1}{2}}J_1^{-1})J_1R_{e,k}^{\frac{1}{2}}$  (21)

$$\begin{bmatrix}
R_k^{\frac{1}{2}} & C_k \hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \\
0 & \rho^{-\frac{1}{2}} \hat{\Sigma}_{k|k-1}^{\frac{1}{2}}
\end{bmatrix} \Theta(k) = \begin{bmatrix}
R_{e,k}^{\frac{1}{2}} & 0 \\
\rho^{-\frac{1}{2}} K_k R_{e,k}^{-\frac{1}{2}} J_1^{-1} & \hat{\Sigma}_{k+1|k}^{\frac{1}{2}}
\end{bmatrix}$$
(22)

Where,

$$R_{k} = R_{k}^{\frac{1}{2}} J_{1} R_{k}^{\frac{T}{2}}, \quad R_{k}^{\frac{1}{2}} = \begin{bmatrix} \rho^{\frac{1}{2}} & 0 \\ 0 & \rho^{\frac{1}{2}} \gamma_{f} \end{bmatrix}, \quad J_{1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \hat{\Sigma}_{k|k-1} = \hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \hat{\Sigma}_{k|k-1}^{\frac{T}{2}}$$

$$R_{e,k} = R_{k} + C_{k} \hat{\Sigma}_{k|k-1} C_{k}^{T}, \quad C_{k} = \begin{bmatrix} H_{k} \\ H_{k} \end{bmatrix}, \quad R_{e,k} = R_{e,k}^{\frac{1}{2}} J_{1} R_{e,k}^{\frac{T}{2}}, \quad \hat{x}_{0|0} = \check{x}_{0}$$
(23)

 $\Theta(\mathbf{k})$  denotes a J-unitary matrix, that is, satisfies

 $\Theta(k) J\ThetaH(k)^T = J$ ,  $J = (J_1 \oplus I)$ , I denotes a unit matrix,

 $K_{\mathtt{k}}(:\,,1)$  denotes a column vector of a first column of the matrix  $K_{\mathtt{k}}\,.$ 

$$R_k = \begin{bmatrix} \rho & 0 \\ 0 & -\rho \gamma_f^2 \end{bmatrix}$$

here,

 $x^*_{k|k}$ : the estimated value of the state  $x_k$  at the time k using the observation signals  $y_0$  to  $y_k$ ,

 $y_k$ : the observation signal,

 $F_k$ : the dynamics of the system,  $F_k = I$  for simplification,

K<sub>s,k</sub>: the filter gain,

 $H_k$ : the observation matrix,

 $\sum^{}_{k|k}$ : corresponding to a covariance matrix of an error of  $x^{}_{k|k}$ ,

 $\Theta(k)$ : the J-unitary matrix, and

Re.k: an auxiliary variable.

a step at which the processing section stores an estimated value of the state  $x_k$  by the hyper  $H_\infty$  filter into the storage section;

a step at which the processing section calculates an existence condition based on the upper limit value  $\gamma_f$  and the forgetting factor  $\rho$  by the obtained observation matrix  $H_i$  or the observation matrix  $H_i$  and the filter gain  $K_{s,i}$ , and

a step at which the processing section decreases the upper limit value  $\gamma_f$  by a factor of  $\Delta\gamma$  and stores the resultant value into the storage section while the existence condition is satisfied in the step of executing the hyper  $H_\infty$  filter,

wherein the  $H_{\infty}$  filter equation is applied to obtain the state estimated value  $x^*_{k|k}=[h^*_1[k],...,\ h^*_N[k]]^T$ , where  $h^*_1[k]$  is the estimated value of impulse response,

a pseudo-echo is estimated by a following expression:

$$\hat{d}_k = \sum_{i=0}^{N-1} \hat{h}_i[k] u_{(k-i)}, \quad k = 0, 1, 2, \cdots$$
 (34)

and an actual echo is cancelled by the obtained pseudo-echo.

Claim 8(previously presented): The system estimation method according to claim 7, wherein the step of executing the hyper  $H_{\infty}$  filter includes:

a step at which the processing section calculates  ${\sum^{\wedge}}_{k+1|k}{}^{1/2}$  by using the expression (22);

a step at which the processing section calculates the filter gain  $K_{s,k}$  based on an initial condition of  $\sum_{k|k-1}^{n}$  and an initial condition of  $C_k$  by using the expression (21);

a step at which the processing section updates a filter equation of the  $H_{\infty}$  filter of the expression (20); and

a step at which the processing section repeatedly executes the step of calculating by using the expression (20), the step of calculating by using the expression (21) and, the step of updating while advancing the time k.

Claim 9 (currently amended): A system estimation method, for a communication system or a sound system or sound field reproduction or noise control, for making state estimation robust and optimizing a forgetting factor  $\rho$  simultaneously in an estimation algorithm, in which

for a state space model expressed by following expressions:

 $x_{k+1} = F_k x_k + G_k w_k$ 

 $y_k = H_k x_k + v_k$ 

 $z_k = H_k x_k$ 

here,

 $x_k$ : a state vector or simply a state,

 $w_k$ : a system noise,

v<sub>k</sub>: an observation noise,

yk: an observation signal,

 $z_k$ : an output signal,

 $F_k$ : dynamics of a system, and

 $G_k$ : a drive matrix,

as an evaluation criterion, a maximum value of an energy gain which indicates a ratio of a filter error to a disturbance including the system noise  $w_k$  and the observation noise  $v_k$  and is weighted with the forgetting factor  $\rho$  is suppressed to be smaller than a term corresponding to a previously given upper limit value  $\gamma_f$ , and

the system estimation method comprises:

a step at which a processing section inputs the upper limit value  $\gamma_f$ , the observation signal  $y_k$  as an input of a filter and a value including an observation matrix  $H_k$  from a storage section or an input section;

a step at which the processing section determines the forgetting factor  $\rho$  relevant to the state space model in accordance with the upper limit value  $\gamma_f$ ; as a following function of  $\gamma_f$ ,

 $\rho = 1 - \chi (\gamma_f)$ 

where  $\chi(\gamma_f)$  denotes a monotonically damping function of  $\gamma_f$  to satisfy  $\chi(1) = 1$  and  $\chi(\infty) = 0$ ,

a step of executing a hyper  $H_{\infty}$  filter at which the processing section reads out an initial value or a value including the observation matrix  $H_{\boldsymbol{k}}$  at a time from the storage section and obtains a filter gain  $K_{s,k}$  by using the forgetting factor  $\rho$  and a gain matrix  $K_k$  and by following expressions:

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k-1|k-1} + \boldsymbol{K}_{s,k} (y_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k-1|k-1})$$

$$\boldsymbol{K}_{s,k} = \boldsymbol{K}_k (:,1) / R_{e,k} (1,1) , \quad \boldsymbol{K}_k = \rho^{\frac{1}{2}} (\overline{\boldsymbol{K}}_k \boldsymbol{R}_{e,k}^{-\frac{1}{2}}) \boldsymbol{R}_{e,k}^{\frac{1}{2}}$$

$$\begin{bmatrix} \boldsymbol{R}_{e,k+1}^{\frac{1}{2}} & \boldsymbol{0} \\ \boldsymbol{K}_{e,k+1} & \boldsymbol{I}_{e,k} & \boldsymbol{C}_{k+1} \tilde{\boldsymbol{L}}_k \boldsymbol{R}_{r,k}^{-\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_{e,k}^{\frac{1}{2}} & \boldsymbol{C}_{k+1} \tilde{\boldsymbol{L}}_k \boldsymbol{R}_{r,k}^{-\frac{1}{2}} \\ \boldsymbol{I}_{e,k} & \boldsymbol{I}_{e,k} & \boldsymbol{I}_{e,k} \end{bmatrix} \boldsymbol{\Theta}(k)$$
(63)

$$\begin{bmatrix}
R_{e,k+1}^{\frac{1}{2}} & 0 \\
\bar{K}_{k+1} & \bar{L}_{k}R_{r,k}^{-\frac{7}{2}} \\
0
\end{bmatrix} R_{e,k+1}^{-\frac{7}{2}} J_{1} \quad \tilde{L}_{k+1}R_{r,k+1}^{-\frac{7}{2}}
\end{bmatrix} = \begin{bmatrix}
R_{e,k}^{\frac{1}{2}} & \check{C}_{k+1}\tilde{L}_{k}R_{r,k}^{-\frac{1}{2}} \\
\bar{L}_{k} & \bar{L}_{k}R_{r,k}^{-\frac{1}{2}}
\end{bmatrix} \Theta(k) \quad (63)$$

here,  $\Theta(k)$  denotes an arbitrary J-unitary matrix, and  $\check{C}_k$ =  $\check{C}_{k+1}\Psi$  is established, where

$$R_{k} = R_{k}^{\frac{1}{2}} J_{1} R_{k}^{\frac{T}{2}}, \quad R_{k}^{\frac{1}{2}} = \begin{bmatrix} \rho^{\frac{1}{2}} & 0 \\ 0 & \rho^{\frac{1}{2}} \gamma_{f} \end{bmatrix}, \quad J_{1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \hat{\Sigma}_{k|k-1} = \hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \hat{\Sigma}_{k|k-1}^{\frac{T}{2}}$$

$$R_{e,k} = R_{k} + C_{k} \hat{\Sigma}_{k|k-1} C_{k}^{T}, \quad C_{k} = \begin{bmatrix} H_{k} \\ H_{k} \end{bmatrix}, \quad R_{e,k} = R_{e,k}^{\frac{1}{2}} J_{1} R_{e,k}^{\frac{T}{2}}, \quad \hat{x}_{0|0} = \check{x}_{0}$$
(23)

$$R_k = \begin{bmatrix} \rho & 0 \\ 0 & -\rho \gamma_f^2 \end{bmatrix}$$

here,

 $x^*_{k}$ : the estimated value of the state  $x_k$  at the time k using the observation signals  $y_0$  to  $y_k$ ,

 $y_k$ : the observation signal,

 $K_{s,k}$ : the filter gain,

 $H_k$ : the observation matrix,

 $\Theta(k)$ : the J-unitary matrix, and

 $R_{e,k}$ : an auxiliary variable.

a step at which the processing section stores an estimated value of the state  $x_k$  by the hyper  $H_\infty$  filter into the storage section;

a step at which the processing section calculates an existence condition based on the upper limit value  $\gamma_f$  and the forgetting factor  $\rho$  by the obtained observation matrix  $H_i$  or the observation matrix  $H_i$  and the filter gain  $K_{s,i}$ , and

a step at which the processing section decreases the upper limit value  $\gamma_f$  by a factor of  $\Delta\gamma$  and stores the resultant value into the storage section while the existence condition is satisfied in the step of executing the hyper  $H_{\infty}$  filter, wherein the  $H_{\infty}$  filter equation is applied to obtain the state estimated value  $x^*_{k|k} = [h^*_1[k], ..., h^*_N[k]]^T$ , where  $h^*_1[k]$  is the estimated value of impulse response,

a pseudo-echo is estimated by a following expression:

$$\hat{d}_k = \sum_{i=0}^{N-1} \hat{h}_i[k] u_{(k-i)}, \quad k = 0, 1, 2, \cdots$$
(34)

and an actual echo is cancelled by the obtained pseudo-echo.

Claim 10 (previously presented): The system estimation method according to claim 9, wherein the step of executing the hyper  $H_{\infty}$  filter includes:

a step at which the processing section calculates  $K_k^-$  based on an initial condition of  $R_{e,\,k+1}$ ,  $R_{r,\,k+1}$  and  $L_{k+1}^-$  by using the expression (63);

a step at which the processing section calculates the filter gain  $K_{s,k}$  based on the initial condition and by using the expression (62);

a step at which the processing section updates a filter equation of the  $H_{\infty}$  filter of the expression (61); and

a step at which the processing section repeatedly executes the step of calculating by using the expression (63), the step of calculating by using the expression (62), and, the step of updating while advancing the time k.

Claim 11 (currently amended): A system estimation method, for a communication system or a sound system or sound field reproduction or noise control, for making state estimation robust and optimizing a forgetting factor  $\rho$  simultaneously in an estimation algorithm, in which

for a state space model expressed by following expressions:

 $x_{k+1} = F_k x_k + G_k w_k$ 

 $y_k = H_k x_k + v_k$ 

 $z_k = H_k x_k$ 

here,

 $x_k$ : a state vector or simply a state,

 $w_k$ : a system noise,

 $v_k$ : an observation noise,

 $y_k$ : an observation signal,

 $z_k$ : an output signal,

 $\ensuremath{\mathbb{F}_k} \colon$  dynamics of a system, and

 $G_k$ : a drive matrix,

as an evaluation criterion, a maximum value of an energy gain which indicates a ratio of a filter error to a disturbance including the system noise  $w_k$  and the observation noise  $v_k$  and is weighted with the forgetting factor  $\rho$  is suppressed to be smaller than a term corresponding to a previously given upper limit value  $\gamma_f$ , and

the system estimation method comprises:

a step at which a processing section inputs the upper limit value  $\gamma_f$ , the observation signal  $y_k$  as an input of a filter and a value including an observation matrix  $H_k$  from a storage section or an input section;

a step at which the processing section determines the forgetting factor  $\rho$  as a following function of  $\gamma_f$  ,  $\rho{=}1{-}\chi\left(\gamma_f\right)$ 

where  $\chi(\gamma_f)$  denotes a monotonically damping function of  $\gamma_f$  to satisfy  $\chi(1)$  = 1 and  $\chi(\infty)$  = 0,

a step of executing a hyper  $H_{\infty}$  filter at which the processing section reads out an initial value or a value including the observation matrix  $H_k$  at a time from the storage section and obtains a filter gain  $K_{s,\,k}$  by using the forgetting factor  $\rho$  and a gain matrix  $K^-_k$  and by following expressions:

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k \hat{x}_{k-1|k-1})$$
(25)

$$K_{s,k} = \rho^{\frac{1}{2}} \overline{K}_k(:,1) / R_{e,k}(1,1)$$
 (26)

$$\begin{bmatrix}
\overline{K}_{k+1} \\
0
\end{bmatrix} = \begin{bmatrix}
0 \\
\overline{K}_k
\end{bmatrix} - \rho^{-\frac{1}{2}} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \tilde{C}_{k+1}^T$$
(27)

$$\tilde{\boldsymbol{L}}_{k+1} = \rho^{-\frac{1}{2}} \tilde{\boldsymbol{L}}_k - \left[ \frac{0}{K_k} \right] R_{e,k}^{-1} \check{\boldsymbol{C}}_{k+1} \tilde{\boldsymbol{L}}_k$$
(28)

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T$$
(29)

$$\boldsymbol{R}_{r,k+1} = \boldsymbol{R}_{r,k} - \tilde{\boldsymbol{L}}_{k}^{T} \boldsymbol{\check{C}}_{k+1}^{T} \boldsymbol{R}_{e,k}^{-1} \boldsymbol{\check{C}}_{k+1} \tilde{\boldsymbol{L}}_{k}$$
(30)

Where.

$$\check{C}_{k+1} = \begin{bmatrix} H_{k+1} \\ \check{H}_{k+1} \end{bmatrix}, \quad \check{H}_{k+1} = [u_{k+1} \ u(k+1-N)] = [u(k+1) \ u_k], \quad \check{H}_1 = [u(1), 0, \dots, 0]$$

$$\boldsymbol{R}_{e,1} = \boldsymbol{R}_1 + \boldsymbol{\check{C}}_1 \boldsymbol{\check{\Sigma}}_{1|0} \boldsymbol{\check{C}}_1^T, \quad \boldsymbol{R}_1 = \begin{bmatrix} \rho & 0 \\ 0 & -\rho \gamma_f^2 \end{bmatrix}, \quad \boldsymbol{\check{\Sigma}}_{1|0} = \operatorname{diag}\{\rho^2, \rho^3, \dots, \rho^{N+2}\}, \quad \rho = 1 - \chi(\gamma_f)$$

$$\tilde{\boldsymbol{L}}_{0} = \begin{bmatrix} 1 & 0 \\ \mathbf{0} & \mathbf{0} \\ 0 & 1 \end{bmatrix} \in \boldsymbol{\mathcal{R}}^{(N+1)\times 2}, \quad \boldsymbol{R}_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & \rho^{-N} \end{bmatrix}, \quad \overline{\boldsymbol{K}}_{0} = \boldsymbol{0}, \quad \hat{\boldsymbol{x}}_{0|0} = \tilde{\boldsymbol{x}}_{0}, \quad \overline{\boldsymbol{K}}_{k} = \rho^{-\frac{1}{2}} \boldsymbol{K}_{k} \quad (31)$$

here,

 $y_k$ : the observation signal,

 $F_k$ : the dynamics of the system,  $F_k$  = I for simplification,

 $H_k$ : the observation matrix,

 $x^*_{k|k}$ : the estimated value of the state  $x_k$  at the time k using the observation signals  $y_0$  to  $y_k$ ,

 $K_{\text{s},k}\colon$  the filter gain, obtained from the gain matrix  $K^{\text{-}}_{k},$  and  $R_{\text{e},k},$   $\tilde{L_{k}}\colon$  an auxiliary variable.

a step at which the processing section stores an estimated value of the state  $x_k$  by the hyper  $H_\infty$  filter into the storage section;

a step at which the processing section calculates an existence condition based on the upper limit value  $\gamma_f$  and the forgetting factor  $\rho$  by the obtained observation matrix  $H_i$  or the observation matrix  $H_i$  and the filter gain  $K_{s,i}$ , and

a step at which the processing section decreases the upper limit value  $\gamma_f$  by a factor of  $\Delta\gamma$  and stores the resultant value into the storage section while the existence condition is satisfied in the step of executing the hyper  $H_{\infty}$  filter, wherein the  $H_{\infty}$  filter equation is applied to obtain the state estimated value  $x^*_{k|k}=[h^*_1[k],...,\ h^*_N[k]]^T$ , where  $h^*_1[k]$  is the estimated value of impulse response,

a pseudo-echo is estimated by a following expression:

$$\hat{d}_k = \sum_{i=0}^{N-1} \hat{h}_i[k] u_{(k-i)}, \quad k = 0, 1, 2, \cdots$$
(34)

and an actual echo is cancelled by the obtained pseudo-echo.

Claim 12 (canceled)

Claim 13 (previously presented): The system estimation method according to claim 7, wherein an estimated value  $z^{v}_{k|k}$  of the output signal is obtained from the state estimated value  $x^{*}_{k|k}$  at the time k by a following expression:

 $z^{v}_{k|k} = H_k x^{h}_{k|k}$ .

Claim 14 (canceled)

Claim 15 (currently amended): A system estimation program product, for a communication system or a sound system or sound field reproduction or noise control, embodied on a computer-readable medium and comprising code that, when executed, causes a computer to make state estimation robust and to optimize a forgetting factor  $\rho$  simultaneously in an estimation algorithm, in which

for a state space model expressed by following expressions:

 $x_{k+1} = F_k x_k + G_k w_k$ 

 $y_k = H_k x_k + v_k$ 

 $z_k = H_k x_k$ 

here,

 $x_k$ : a state vector or simply a state,

 $w_k$ : a system noise,

v<sub>k</sub>: an observation noise,

 $y_k$ : an observation signal,

 $z_k$ : an output signal,

 $F_k$ : dynamics of a system, and

 $G_k$ : a drive matrix,

as an evaluation criterion, a maximum value of an energy gain which indicates a ratio of a filter error to a disturbance including the system noise  $w_k$  and the observation noise  $v_k$  and is weighted with the forgetting factor  $\rho$  is suppressed to be smaller than a term corresponding to a previously given upper limit value  $\gamma_f$ , and

the system estimation program causes the computer to execute:

a step at which a processing section inputs the upper limit value  $\gamma_f$ , the observation signal  $y_k$  as an input of a filter and a value including an observation matrix  $H_k$  from a storage section or an input section;

a step at which the processing section determines the forgetting factor  $\rho$  as a following function of  $\gamma_f$  ,  $\rho{=}1{-}\chi\left(\gamma_f\right)$ 

where  $\rho$   $(\gamma_f)$  denotes a monotonically damping function of  $\gamma_f$  to satisfy  $\chi(1)$  = 1 and  $\chi(\infty)$  = 0,

a step of executing a hyper  $H_{\infty}$  filter at which the processing section reads out an initial value or a value including the observation matrix  $H_k$  at a time from the storage section and obtains a filter gain  $K_{s,\,k}$  by using the forgetting factor  $\rho$  and a gain matrix  $K^-_k$  and by following expressions:

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k \hat{x}_{k-1|k-1})$$
(25)

$$K_{s,k} = \rho^{\frac{1}{2}} \overline{K}_k(:,1) / R_{e,k}(1,1)$$
 (26)

$$\frac{\boldsymbol{K}_{s,k} = \rho^{-1} \boldsymbol{K}_{k}(\cdot, 1) / \operatorname{Re}_{,k}(1, 1)}{\left[\begin{array}{c} \boldsymbol{\overline{K}}_{k+1} \\ \boldsymbol{0} \end{array}\right] = \left[\begin{array}{c} \boldsymbol{0} \\ \boldsymbol{\overline{K}}_{k} \end{array}\right] - \rho^{-\frac{1}{2}} \tilde{\boldsymbol{L}}_{k} \boldsymbol{R}_{r,k}^{-1} \tilde{\boldsymbol{L}}_{k}^{T} \boldsymbol{C}_{k+1}^{T} \tag{27}$$

$$\tilde{\boldsymbol{L}}_{k+1} = \rho^{-\frac{1}{2}} \tilde{\boldsymbol{L}}_k - \begin{bmatrix} \mathbf{0} \\ \overline{\boldsymbol{K}}_k \end{bmatrix} \boldsymbol{R}_{e,k}^{-1} \check{\boldsymbol{C}}_{k+1} \tilde{\boldsymbol{L}}_k$$
 (28)

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T$$
(29)

$$\boldsymbol{R}_{r,k+1} = \boldsymbol{R}_{r,k} - \tilde{\boldsymbol{L}}_{k}^{T} \boldsymbol{\check{C}}_{k+1}^{T} \boldsymbol{R}_{e,k}^{-1} \boldsymbol{\check{C}}_{k+1} \tilde{\boldsymbol{L}}_{k}$$
(30)

Where,

$$\check{\boldsymbol{C}}_{k+1} = \begin{bmatrix} \check{\boldsymbol{H}}_{k+1} \\ \check{\boldsymbol{H}}_{k+1} \end{bmatrix}, \quad \check{\boldsymbol{H}}_{k+1} = [\boldsymbol{u}_{k+1} \ u(k+1-N)] = [u(k+1) \ \boldsymbol{u}_k], \quad \check{\boldsymbol{H}}_1 = [u(1), 0, \dots, 0]$$

$$\boldsymbol{R}_{e,1} = \boldsymbol{R}_1 + \boldsymbol{\check{C}}_1 \boldsymbol{\check{\Sigma}}_{1|0} \boldsymbol{\check{C}}_1^T, \quad \boldsymbol{R}_1 = \begin{bmatrix} \rho & 0 \\ 0 & -\rho \gamma_f^2 \end{bmatrix}, \quad \boldsymbol{\check{\Sigma}}_{1|0} = \operatorname{diag}\{\rho^2, \rho^3, \dots, \rho^{N+2}\}, \quad \rho = 1 - \chi(\gamma_f)$$

$$\tilde{\boldsymbol{L}}_{0} = \begin{bmatrix} 1 & 0 \\ \mathbf{0} & \mathbf{0} \\ 0 & 1 \end{bmatrix} \in \boldsymbol{\mathcal{R}}^{(N+1)\times 2}, \quad \boldsymbol{R}_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & \rho^{-N} \end{bmatrix}, \quad \overline{\boldsymbol{K}}_{0} = \boldsymbol{0}, \quad \hat{\boldsymbol{x}}_{0|0} = \tilde{\boldsymbol{x}}_{0}, \quad \overline{\boldsymbol{K}}_{k} = \rho^{-\frac{1}{2}} \boldsymbol{K}_{k} \quad (31)$$

here,

 $y_k$ : the observation signal,

 $F_k$ : the dynamics of the system,  $F_k = I$  for simplification,

 $H_k$ : the observation matrix,

 $x^*_{k|k}$ : the estimated value of the state  $x_k$  at the time k using the observation signals  $y_0$  to  $y_k$ ,

 $K_{s,k}$ : the filter gain, obtained from the gain matrix  $K_k^-$ , and  $R_{e,k}$ ,  $L_k^-$ : an auxiliary variable.

a step at which the processing section stores an estimated value of the state  $x_k$  by the hyper  $H_\infty$  filter into the storage section;

a step at which the processing section calculates an existence condition based on the upper limit value  $\gamma_f$  and the forgetting factor  $\rho$  by the obtained observation matrix  $H_i$  or the observation matrix  $H_i$  and the filter gain  $K_{s,i}$ , and

a step at which the processing section decreases the upper limit value  $\gamma_f$  by a factor of  $\Delta\gamma$  and stores the resultant value into the storage section while the existence condition is satisfied in the step of executing the hyper  $H_\infty$  filter;

wherein the  $H_{\infty}$  filter equation is applied to obtain the state estimated value  $x^*_{k|k}=[h^*_{1}[k],...,\ h^*_{N}[k]]^{T}$ , where  $h^*_{i}[k]$  is the estimated value of impulse response,

a pseudo-echo is estimated by a following expression:

$$\hat{d}_k = \sum_{i=0}^{N-1} \hat{h}_i[k] u_{(k-i)}, \quad k = 0, 1, 2, \cdots$$
(34)

and an actual echo is cancelled by the obtained pseudo-echo.

Claim 16 (currently amended): A computer readable recording medium recording a system estimation program product, for a communication system or a sound system or sound field reproduction or noise control, embodied on a computer readable medium and comprising code that, when executed, causes a computer to make state estimation robust and to

optimize a forgetting factor  $\rho$  simultaneously in an estimation algorithm, in which

for a state space model expressed by following expressions:

 $x_{k+1} = F_k x_k + G_k w_k$ 

 $y_k = H_k x_k + v_k$ 

 $z_k = H_k x_k$ 

here,

 $x_k$ : a state vector or simply a state,

 $w_k$ : a system noise,

vk: an observation noise,

 $y_k$ : an observation signal,

 $z_k$ : an output signal,

 $F_k$ : dynamics of a system, and

 $G_k$ : a drive matrix,

as an evaluation criterion, a maximum value of an energy gain which indicates a ratio of a filter error to a disturbance including the system noise  $w_k$  and the observation noise  $v_k$  and is weighted with the forgetting factor  $\rho$  is suppressed to be smaller than a term corresponding to a previously given upper limit value  $\gamma_f,$  and

the computer readable recording medium recording the system estimation program causes the computer to execute:

a step at which a processing section inputs the upper limit value  $\gamma_f$ , the observation signal  $y_k$  as an input of a filter and a value including an observation matrix  $H_k$  from a storage section or an input section;

a step at which the processing section determines the forgetting factor  $\rho\, relevant$  to the state space model in accordance as a following function of  $\gamma_f,$ 

 $\rho = 1 - \chi (\gamma_f)$ 

where  $\chi(\gamma_f)$  denotes a monotonically damping function of  $\gamma_f$  to satisfy  $\chi(1)$  = 1 and  $\chi(\infty)$  = 0,

a step of executing a hyper  $H_{\infty}$  filter at which the processing section reads out an initial value or a value including the observation matrix  $H_k$  at a time from the storage section and obtains a filter gain  $K_{s,k}$  by using the forgetting factor  $\rho$  and a gain matrix  $K_k^-$  and by following expressions:

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k \hat{x}_{k-1|k-1})$$

$$K_{s,k} = \rho^{\frac{1}{2}} \overline{K}_k(:,1) / R_{e,k}(1,1)$$

$$\begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T$$

$$\hat{L}_{k+1} = \rho^{-\frac{1}{2}} \tilde{L}_k - \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k$$

$$R_{e,k} = R_{e,k} - \check{C}_{k+1} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T$$

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T$$

$$R_{r,k+1} = R_{r,k} - \tilde{L}_k^T \check{C}_{k+1}^T R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k$$

$$\tilde{C}_{k+1} = \begin{bmatrix} \check{H}_{k+1} \\ \check{H}_{k+1} \end{bmatrix}, \quad \check{H}_{k+1} = [u_{k+1} \ u(k+1-N)] = [u(k+1) \ u_k], \quad \check{H}_1 = [u(1), 0, \dots, 0]$$

$$R_{e,1} = R_1 + \check{C}_1 \check{\Sigma}_{1|0} \check{C}_1^T, \quad R_1 = \begin{bmatrix} \rho & 0 \\ 0 & -\rho \gamma_f^2 \end{bmatrix}, \quad \check{\Sigma}_{1|0} = \text{diag}\{\rho^2, \rho^3, \dots, \rho^{N+2}\}, \quad \rho = 1 - \chi(\gamma_f)$$

$$\tilde{L}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{(N+1) \times 2}, \quad R_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & -\rho^{-N} \end{bmatrix}, \quad \overline{K}_0 = 0, \quad \hat{x}_{0|0} = \check{x}_0, \quad \overline{K}_k = \rho^{-\frac{1}{2}} K_k$$
 (31)

here,

 $y_k$ : the observation signal,

 $F_k$ : the dynamics of the system,  $F_k = I$  for simplification,

 $H_k$ : the observation matrix,

 $x^*_{k|k}$ : the estimated value of the state  $x_k$  at the time k using the observation signals  $y_0$  to  $y_k$ ,

 $K_{s,k} \colon$  the filter gain, obtained from the gain matrix  $K^-_{k},$  and  $R_{e,k},$   $\tilde{L_k} \colon$  an auxiliary variable.

a step at which the processing section stores an estimated value of the state  $x_k$  by the hyper  $H_\infty$  filter into the storage section;

a step at which the processing section calculates an existence condition based on the upper limit value  $\gamma_f$  and the forgetting factor  $\rho$  by the obtained observation matrix  $H_i$  or the observation matrix  $H_i$  and the filter gain  $K_{s,i}$ , and

a step at which the processing section decreases the upper limit value  $\gamma_f$  by a factor of  $\Delta\gamma$  and stores the resultant value into the storage section while the existence condition is satisfied in

the step of executing the hyper  $H_{\infty}$  filter,

wherein the  $H_{\infty}$  filter equation is applied to obtain the state estimated value  $x^*_{k|k}=[h^*_{1}[k],...,\ h^*_{N}[k]]^{T}$ , where  $h^*_{i}[k]$  is the estimated value of impulse response,

a pseudo-echo is estimated by a following expression:

$$\hat{d}_k = \sum_{i=0}^{N-1} \hat{h}_i[k] u_{(k-i)}, \quad k = 0, 1, 2, \cdots$$
(34)

and an actual echo is cancelled by the obtained pseudo-echo.

Claim 17 (currently amended): A system estimation device, for a communication system or a sound system or sound field reproduction or noise control, for making state estimation robust and optimizing a forgetting factor  $\rho$  simultaneously in an estimation algorithm, in which

for a state space model expressed by following expressions:

 $x_{k+1} = F_k x_k + G_k w_k$ 

 $y_k = H_k x_k + v_k$ 

 $z_k = H_k x_k$ 

here,

 $x_k$ : a state vector or simply a state,

 $w_k$ : a system noise,

 $v_k$ : an observation noise,

 $y_k$ : an observation signal,

 $z_k$ : an output signal,

 $F_k$ : dynamics of a system, and

 $G_k$ : a drive matrix,

as an evaluation criterion, a maximum value of an energy gain which indicates a ratio of a filter error to a disturbance including the system noise  $w_k$  and the observation noise  $v_k$  and is weighted with the forgetting factor  $\rho$  is suppressed to be smaller than a term corresponding to a previously given upper limit value  $\gamma_f$ , and

the system estimation device comprises:

a processing section to execute the estimation algorithm; and

a storage section to which reading and/or writing is performed by the processing section and which stores respective observed values, set values, and estimated values relevant to the state space model,

further comprising:

a means at which the processing section inputs the upper limit value  $\gamma_f$ , the observation signal  $y_k$  as an input of a filter and a value including an observation matrix  $H_k$  from the storage section or an input section;

a means at which the processing section determines the forgetting factor  $\rho$  as a following function of  $\gamma_f$  ,  $\rho{=}1{-}\chi\left(\gamma_f\right)$ 

where  $\chi(\gamma_f)$  denotes a monotonically damping function of  $\gamma_f$  to satisfy  $\chi(1)$  = 1 and  $\chi(\infty)$  = 0,

a means of executing a hyper  $H_{\infty}$  filter at which the processing section reads out an initial value or a value including the observation matrix  $H_k$  at a time from the storage section and obtains a filter gain  $K_{s,\,k}$  by using the forgetting factor  $\rho$  and a gain matrix  $K^-_k$  and by following expressions:

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k-1|k-1} + \boldsymbol{K}_{s,k}(y_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k-1|k-1})$$
(25)

$$K_{s,k} = \rho^{\frac{1}{2}} \overline{K}_k(:,1) / R_{e,k}(1,1)$$
 (26)

$$\frac{\overline{K}_{s,k} = \rho \cdot \overline{K}_{k}(t,1) / \overline{K}_{e,k}(t,1)}{\left[\overline{K}_{k+1} \right] = \left[\frac{0}{\overline{K}_{k}}\right] - \rho^{-\frac{1}{2}} \tilde{L}_{k} R_{r,k}^{-1} \tilde{L}_{k}^{T} \breve{C}_{k+1}^{T} \tag{27}$$

$$\tilde{L}_{k+1} = \rho^{-\frac{1}{2}} \tilde{L}_k - \left[ \frac{0}{K_k} \right] R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k$$
(28)

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T$$
(29)

$$\boldsymbol{R}_{r,k+1} = \boldsymbol{R}_{r,k} - \tilde{\boldsymbol{L}}_{k}^{T} \tilde{\boldsymbol{C}}_{k+1}^{T} \boldsymbol{R}_{e,k}^{-1} \tilde{\boldsymbol{C}}_{k+1} \tilde{\boldsymbol{L}}_{k}$$
(30)

Where,

$$\check{C}_{k+1} = \begin{bmatrix} \check{H}_{k+1} \\ \check{H}_{k+1} \end{bmatrix}, \quad \check{H}_{k+1} = [u_{k+1} \ u(k+1-N)] = [u(k+1) \ u_k], \quad \check{H}_1 = [u(1), 0, \dots, 0]$$

$$m{R}_{e,1} = m{R}_1 + m{\check{C}}_1 m{\check{\Sigma}}_{1|0} m{\check{C}}_1^T, \quad m{R}_1 = \left[ egin{array}{ccc} 
ho & 0 \ 0 & -
ho \gamma_f^2 \end{array} 
ight], \quad m{\check{\Sigma}}_{1|0} = \mathrm{diag}\{
ho^2, 
ho^3, \ldots, 
ho^{N+2}\}, \quad 
ho = 1 - \chi(\gamma_f)$$

$$\tilde{\boldsymbol{L}}_{0} = \begin{bmatrix} 1 & 0 \\ \mathbf{0} & \mathbf{0} \\ 0 & 1 \end{bmatrix} \in \boldsymbol{\mathcal{R}}^{(N+1)\times 2}, \quad \boldsymbol{R}_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & \rho^{-N} \end{bmatrix}, \quad \overline{\boldsymbol{K}}_{0} = \boldsymbol{0}, \quad \hat{\boldsymbol{x}}_{0|0} = \tilde{\boldsymbol{x}}_{0}, \quad \overline{\boldsymbol{K}}_{k} = \rho^{-\frac{1}{2}} \boldsymbol{K}_{k} \quad (31)$$

here,

yk: the observation signal,

 $F_k$ : the dynamics of the system,  $F_k = I$  for simplification,

 $H_k$ : the observation matrix,

 $x^*_{k|k}$ : the estimated value of the state  $x_k$  at the time k using the observation signals  $y_0$  to  $y_k$ ,

 $K_{s,k}$ : the filter gain, obtained from the gain matrix  $K_k^-$ , and  $R_{e,k}$ ,  $L_k^-$ : an auxiliary variable.

a means at which the processing section stores an estimated value of the state  $x_k$  by the hyper  $H_\infty$  filter into the storage section;

a means at which the processing section calculates an existence condition based on the upper limit value  $\gamma_f$  and the forgetting factor  $\rho$  by the obtained observation matrix  $H_i$  or the observation matrix  $H_i$  and the filter gain  $K_{s,i}$ , and

a means at which the processing section decreases the upper limit value  $\gamma_f$  by a factor of  $\Delta \gamma$  and stores the resultant

value into the storage section while the existence condition is satisfied in the means of executing the hyper  $H_{\infty}$  filter,

wherein the  $H_{\infty}$  filter equation is applied to obtain the state estimated value  $x^*_{k|k}=[h^*_{1}[k],...,\ h^*_{N}[k]]^{T}$ , where  $h^*_{i}[k]$  is the estimated value of impulse response,

a pseudo-echo is estimated by a following expression:

$$\hat{d}_k = \sum_{i=0}^{N-1} \hat{h}_i[k] u_{(k-i)}, \quad k = 0, 1, 2, \cdots$$
 (34)

and an actual echo is cancelled by the obtained pseudo-echo.

Claim 18 (previously presented): The system estimation method according to claim 9, wherein the processing section calculates the existence condition in accordance with a following expression:

$$-\rho \hat{\Xi}_{i} + \rho \gamma_{f}^{2} > 0, \quad i = 0, \dots, k$$

$$\rho = 1 - \gamma_{f}^{2}, \quad \hat{\Xi}_{i} = \frac{\rho \boldsymbol{H}_{i} \boldsymbol{K}_{s,i}}{1 - \boldsymbol{H}_{i} \boldsymbol{K}_{s,i}}, \quad \rho = 1 - \chi(\gamma_{f})$$

$$(18)$$

where the forgetting factor  $\rho$  and the upper limit value  $\gamma_f$  have a following relation:

 $0<\rho=1-\chi(\gamma_f)\leq 1$ , where  $\chi(\gamma_f)$  denotes a monotonically damping function of  $\gamma_f$  to satisfy  $\chi(1)=1$  and  $\chi(\infty)=0$ .

Claim 19 (previously presented): The system estimation method according to claim 9, wherein an estimated value  $z^{v}_{k|k}$  of the output signal is obtained from the state estimated value  $x^{*}_{k|k}$  at the time k by a following expression:  $z^{v}_{k|k} = H_{k}x^{*}_{k|k}$ .

Claim 20 (canceled)

Claim 21 (previously presented): The system estimation method according to claim 11, wherein the processing section calculates the existence condition in accordance with a following expression:

$$-\rho \hat{\Xi}_{i} + \rho \gamma_{f}^{2} > 0, \quad i = 0, \dots, k$$

$$\rho = 1 - \gamma_{f}^{2}, \quad \hat{\Xi}_{i} = \frac{\rho \boldsymbol{H}_{i} \boldsymbol{K}_{s,i}}{1 - \boldsymbol{H}_{i} \boldsymbol{K}_{s,i}}, \quad \rho = 1 - \chi(\gamma_{f})$$

$$(18)$$

where the forgetting factor  $\rho$  and the upper limit value  $\gamma_f$  have a following relation:

 $0<\rho=1-\chi(\gamma_f)\leq 1$ , where  $\chi(\gamma_f)$  denotes a monotonically damping function of  $\gamma_f$  to satisfy  $\chi(1)=1$  and  $\chi(\infty)=0$ .

Claim 22 (previously presented): The system estimation method according to claim 11, wherein an estimated value  $z^{v}_{k|k}$  of the output signal is obtained from the state estimated value  $x^{\wedge}_{k|k}$  at the time k by a following expression:  $z^{v}_{k|k} = H_{k}x^{\wedge}_{k|k}$ .

Claim 23 (canceled)

## Remarks

Claims 2-3, 7-11, 13, 15-19, and 21-22 are now pending for consideration of the Supervisory Primary Examiner.

Enclosed is a clean set of pending claims as amended as per the Supervisory Patent Examiner's request.

Respectfully submitted,

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